

# Whitehead's Trilogy and the Curvature of Spacetime

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**ABSTRACT.** We outline the basic ideas of Whitehead's unified theory of Gravitation and Electromagnetism; summarize the evidence that Spacetime is curved and conclude that it is not.

## 1.

## 2. Whitehead's Trilogy and the Curvature of Spacetime

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**2.1. I. Introduction.** In the three years 1919-1922, Alfred North Whitehead (ANW) published three profound books<sup>1</sup> unifying Gravitation and Electromagnetism. This Trilogy, in my opinion, contains the most important contribution to fundamental physical theory by any one individual since Newton. As far as I have discovered, it has hardly been noticed in the past 40 years by mainstream Physicists who have unfortunately allowed Whitehead to be pre-empted by philosophers even though he was professionally occupied as a Mathematical Physicist until he was 63!

The first two volumes of the Trilogy are introductory to the third entitled *Principle of Relativity with applications to Physical Science (PRel)*. In this, ANW shows that there is a fundamental logical flaw in the General Theory of Relativity (GTR)<sup>2</sup> such that in its current form, any physical theory based on GTR is inevitably flawed. He insists that experiment and logic are the ultimate tests of a physical theory and that there is no *valid* evidence to indicate that Spacetime (ST) is curved. This claim cannot be lightly dismissed since it was Whitehead who with his former "pupil", Bertrand Russell, wrote the famous three-volume *Principia Mathematica* in Symbolic Logic establishing themselves as the two greatest Logicians of the past 200 years.

Since I taught GTR enthusiastically at the University of Toronto for 8 years, it is my duty to admit my error and explain how I and most Mathematical Physicists were misled by Eddington.

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*Key words and phrases.* impetus, relativity, gravitation.

We shall see that the essential problem is that of *definition of units*. It was Eddington who got us into the difficulty in 1920. In my Concluding Observations I shall point to an idea from his *Fundamental Theory* which suggests that by 1947 he realized that defining units is an important non-trivial difficulty.

The Trilogy is perhaps the high-point in a series of nine<sup>3</sup> major works ANW published between 1905 and 1929 providing us with insight into his attempt to define concepts and words with which to free himself and us from the brain-washing in which he lived, and we still live, about the meaning of reality, space, time and matter, ideas for example which we inherited about substance from Aristotle and about space or time from Newton. His 1926-27 Gifford Lectures, sub-titled *An Essay in Cosmology* is perhaps the only Metaphysical framework proposed by a scientist with an understanding of the Special and General Theories of Relativity and of Planck's Quantum Theory.

My object in the present paper is to:

- introduce physicists to the life and work of Whitehead, Sec. II;
- outline the content of *The Principle of Relativity* (PRel), Sec. III;
- present ANW's basic criticism of GTR, which, to my mind, has never been rebutted or properly appreciated, even by GTR specialists, Sec. IV;
- propose a significant relevant research problem, Secs. VI & VIII;
- summarize some of the rather radical implications of the acceptance of ANW's observation.

**2.2. II. Life and Work (1861-1947).** His career is best characterized as that of a Mathematical Physicist<sup>4</sup> seeking total understanding of his own physical nature and its place in the ongoing process which we experience as human beings and call the Universe.

Alfred was born on February 15, 1861 in a family of Anglican clergy, school masters and local Administrators. His father was a priest, a school master and an honorary Canon of Canterbury Cathedral which is the spiritual centre of the Anglican Communion throughout the world.

In 1876, Whitehead entered Sherborne School which, according to tradition, had been attended by King Alfred the Great and of which Whitehead became Senior Prefect responsible for behaviour and discipline. He won a scholarship to Trinity College, Cambridge - majored in Mathematics and graduated in 1884 as Fourth Wrangler in the demanding Tripos Examination. He was soon named a Fellow of Trinity College partly as a result of an Essay on Maxwell's Theory which was still quite novel. As a Fellow his basic task was the preparation of College students to master Mathematical Analysis, Analytical Dynamics of Lagrange and Hamilton together with Newton's theory of Gravitation and Maxwell's theory of Electrodynamics. The latter was his favourite topic for university Lectures.

Among his early mathematical writings were papers on approximating the Motion of viscous fluids, Group theory, and Geodesic Geometry. A major book entitled *Universal Algebra* (1898) made him the first winner of the international Lobachevsky Award. This book contained the first clear exposition in English of Grassmann's algebra which now plays a key role in theoretical physics. It is really the language of the QM of fermions and of exterior differential forms.

From 1905 to 1912, Whitehead's big project, together with Bertrand Russell, was writing the three volume treatise, *Principia Mathematica*, on the foundations of Mathematics.

He decided in 1912 to move from Cambridge and was soon established in the University of London as Professor of Applied Mathematics at the *Imperial College of Science and Technology* which vies with MIT in the USA, and ETH in Switzerland as among the truly great Engineering schools.

From there, when he was 63, he was invited to become a Professor of Philosophy in Harvard. He made an enormous impact on philosophical thought epitomized in his *PROCESS and REALITY* which soon became the basis of a totally new important School of philosophy generally referred to as *Process Philosophy* for which "process" is an ultimate category.

For physicists this implies that we must learn to think of the basic constituents of physical reality as "events" not as bits of red or black Aristotelian "substance". To imitate a famous phrase<sup>5</sup> of Reinhold Niebhuur, this will require a "trans-imagining of our imagination"! I am far from accomplishing this even though I have known for more than sixty years that this is required of the physics community. I was 20 years old<sup>6</sup> when, accidentally, I first tackled PRel.

From 1924 until his death, Whitehead wrote prolifically, conducted Graduate Seminars at Harvard and lectured widely /throughout the USA applying his deep intelligence to all the problems of mankind. For example, I found his books on *The Function of Reason* and *Adventures of Ideas* especially interesting, even exciting. For more detailed information about ANW's life and writings as Mathematical Physicist and as Philosopher I advise the reader to consult the definitive biography by Lowe<sup>7</sup>.

### 2.3.

**2.4. III. Whitehead's *Principle of Relativity*.(PRel).** Throughout this Article I use the abbreviation "PRel" to denote Whitehead's book<sup>1</sup>, *The Principle of Relativity with Applications to Physical Science*, and PNK and CN for the other<sup>1</sup> two books in the Trilogy.

There was so little interest in PRel that the Cambridge University Press never reprinted the 1922 First Edition. Fortunately after the Copyright lapsed, Dover and Phoenix, Publishers, recently made the original text available. There is now even a paperback Edition so anyone interested has easy access to it. I entertain three reasons why Physicists have dismissed PRel: (1) the Einstein Euphoria<sup>8</sup> which has been thoroughly documented, (2) the disdain<sup>9</sup> that Physicists had for philosophy until quite recently, and (3) admittedly, PRel is not an easy read.

A paper<sup>10</sup> in *Latex* to which I refer as (*JLS*), contains a brief Introduction and an Appendix on the Solar Limb Effect by myself, but, chiefly, three lectures by *John Leighton Synge* which reproduce in current notation the contents of a mathematical exposition of those aspects of PRel which are pertinent to the claims by Einstein to predict the Advance of the Perihelion of Mercury and the Bending of star-light passing close to the Sun.

Unfortunately Synge replaces ANW's notation  $J_{ij}$  for Whitehead's gravitational impetus by  $g_{ij}$  causing great confusion for any reader familiar with current expositions of GTR. He also, reveals, almost proudly, that he did not read Part I seriously. He therefore missed the main point of PRel.

The book is divided into three Parts:

- I. General Principles, Chaps. 1-4
- II. Physical Applications, Chaps. 5-17
- III. Elementary Theory of Tensors, Chaps. 18-24

As Whitehead explained in his Preface, Part I assembles two formal lectures which he gave in Bryn Mawr and the Royal Society of Edinburgh together with material to students of The Imperial College. I found that, as he feared, this and haste did cause some incoherence and lack of desirable transition material at certain points.

Whitehead sent PRel to the Publisher near the beginning of 1922 so knew perfectly well that the London and the New York Times had already elevated Einstein to the status of the untouchable Icon he became. So, to justify his temerity in criticising GTR in PRel and CN, he praised Einstein to the skies adding the caveat: "The worst homage we can pay to genius is to accept uncritically the formulations of truths which we owe to it" - PRel p.88.

To no avail. The Einstein Euphoria swept him aside - like a man in a canoe defying the tsunami which devastated S-E Asia! The fact pointed out by Whitehead that there is an obvious LOGICAL flaw in GTR has, as far as I have discovered, never been directly confronted in the Physics literature - probably, not even noticed! However, it has been recently analyzed by two philosophers<sup>11</sup>.

In Part I of PRel, ANW announced *four* possible self-consistent unified theories of Gravitation and Electromagnetism each of which satisfies the basic principles which guided Einstein to GTR, but assumes that ST is flat Minkowskian. At least two, (i) and (iv), predict the observed values for the Advance of the perihelion of Mercury and the Bending of Light in the field of the sun. It was on the basis of these two results that in 1920 Eddington had proclaimed GTR as correct.

I found that Part I contained some of Whitehead's most challenging writing because in it he explicitly defines ideas which go beyond our traditional physical concepts and presupposes that the reader has intimate understanding of PNK and CN<sup>1</sup>.

On PRel p. 86/7, he defines four plausible distinct theories of Gravitation, invariant with respect to Lorentz Transformations, each of which assumes that ST is Minkowskian but differ by what he calls Law (i), (ii), (iii), or (iv). In each case the "Law" specifies how the "gravitational *IMPETUS*" is defined by a quadratic form  $J_{ij}dx^i dx^j$ . The first three Laws involve solving a coupled system of ten second order partial differential equations. They were obtained by a relatively simple variation on Einstein's procedure for obtaining the Field Equations of GTR.

In all of them, ST is Minkowskian with metric determined by a second order covariant Tensor,  $\omega_{ij}$ , for which orthogonal cartesian coordinates may be chosen such that  $\omega_{ij} = \omega_i^2 \delta_{ij}$ , with  $\omega_i^2 = 1$  for  $i = 1, 2, \text{ or } 3$  and  $-1$  for  $i = 4$ . In other words, the familiar metric for Minkowsky Spacetime.

These four theories of gravity together with his formula for the interaction of the gravitational and electromagnetic fields constitute what I am calling his four possible Unified Field Theories. Theory (i) is closest to GTR while assuming that space is flat.

Of course, GTR also has a metric defined by a second order covariant tensor which is almost always denoted by  $g_{ij}$ . According to ANW, Einstein's crucial error is to identify  $g_{ij}$  with  $J_{ij}$ . As he says on PRel, p. 83 "By identifying the potential

mass impetus of a kinematic element with a spatio-temporal measurement Einstein, in my opinion, leaves the whole antecedent theory of measurement in confusion when it is confronted with the actual conditions of our perceptual knowledge.”

For Einstein, the key functions,  $g_{ij}$ , are defined as the solution of 10 PDE's with initial conditions determined by the circumambient massive particles and therefore, the whole massive Universe!. According to Whitehead this is the crucial point at which Einstein introduces a *vicious circle* into the basis of GTR which is discussed in Section V.

ANW is frequently difficult to understand because he takes seriously the changes in our classical mode of thinking about reality forced by the Special Theory of Relativity. There is not space here to summarize this 24-year-long development which began in 1905 with a paper *On Mathematical Concepts of the Material World*, followed in 1916 by *Space, Time, and Relativity*, in 1919-22 by the Trilogy *PNK*, *CN*, *PR* and culminated in 1929 by his famous metaphysical treatise, *Process and Reality - an Essay in Cosmology*. In these he introduced several new concepts and new words. But his mind never rested so the exact meaning of some words changed, causing trouble for us who struggle to understand his basic thinking!

A key concept in Whitehead's physics is *IMPETUS*. This has a role in Whiteheadian Dynamics analogous to "action" in classical dynamics. If we denote "action" by  $A$  then classically we would think of the Principle of Least Action as requiring that a particle  $P$  which goes from a point  $E$  to another point  $F$  follows the path which minimizes the integral of  $dA$  along its path. In my old unreformed mode, I would imagine the particle as a minute very solid ball, probably black (but perhaps red, depending on my mood!) zooming along a Feynmann diagram. Classically, I think of the Action as a property of the *instantaneous* little particle. For ANW only events have reality so potential Impetus is ascribable to finite portions of the path of my particle, so  $\sqrt[2]{dJ^2}$  to the *interval* ( $dX^i$ ). Thus, not to the instantaneous particle as such but to portions of its history.

*Impetus*,  $I$ , is defined implicitly in Part I on p. 79 of PRel in equation (9) which reads

$$dI = M\sqrt[2]{dJ^2} + c^{-1}EdF$$

where  $\sqrt[2]{dJ^2}$  is the potential gravitational mass impetus and  $dF$  is the potential electromagnetic impetus for the interval  $dX$ , ( $X, X + dX$ ). Thus impetus involved the forces known to physicists in 1922.

In the first of the Trilogy, on the *Principles of Natural Knowledge(PNK)*, 1919 ANW pursued his attempt to find words and enunciate new concepts with which to incorporate into our thinking the implications of Special Relativity and Quantum Theory as they gradually emerged and were modified during the period 1890 - 1924. This is a very difficult task since both he and we have unconsciously been "brain-washed" by Aristotelian ideas of 'substance' and Newton's view of the relation of 'time' and 'space'. His attempt began in 1905<sup>3</sup> and culminated in 1927 with his great metaphysical treatise. So persons like myself or Synge, Schild and Will, and many philosophers who think we have understood Whitehead's theory are likely wrong since we have aimed at a moving target!

It is often alleged that it is difficult to understand his writing because ANW is confused and his style is bad. In fact, I find that his style is normally clear even dramatic as is illustrated by his description of the 1919 Meeting of the Royal

Society at which Eddington announced the results of the Observation, for which he was responsible, of the bending of light by the sun. This was proclaimed by Eddington and accepted, by the London and New York Times, as "proof" that Einstein's GTR was correct.

ANW had no hesitation in agreeing that Eddington's Observation showed that Newton's theory of gravitation was inadequate but he did not accept GTR as finally correct because he quickly noted a basic Logical error and also, as is clear from many assertions particularly in his discussion with Lucien Price<sup>12</sup>, he did not think that human beings, *including himself*, are capable of formulating in human language ultimate truth. *This may be his basic difference from Einstein.*

Hitherto I have seen no clear written evidence of *when* he realized the error in GTR but from Chapter 8 of CN and my estimate of the time needed to invent and calculate the consequences of his Law(iv) as set forth in PRel, I consider sometime in 1917 as most likely. Since on his instructions all his personal files were destroyed upon his death we shall probably never know the exact date but I shall now speculate how he was led to Theory (iv).

As soon as he saw the text of Einstein's 1915 paper, ANW saw the logical flaw in it and also thought that the Einstein's prediction of the observed value for the Advance of the Perihelion of Mercury, which was an accepted fact among astronomers, was puzzling since he knew that GTR is illogical. He knew, as did all the Cambridge mathematical physicists, that Maxwell's Theory of Electromagnetism was invariant with respect to Lorentz Transformations. Law(iv) is probably the simplest Lorentz invariant modification of Newton's Inverse Square law for gravity that could be devised.

It is clear from CN Ch. 8 that he had in mind the basic structure of his final theory by late 1919 or early 1920 and knew that it gave exactly the same *formula* for the Advance of the perihelion as Einstein claimed for GTR. He immediately set to work to find a theory consistent with his basic Physical ideas set forth in PNK, CN and Part I of PRel and consistent with observations. In fact he found FOUR logically valid theories, invariant with respect to Lorentz transformations, and Theories(i) and (iv) predict the observed values for the perihelion and the bending of light. The "simplest" of these is what he calls Law (iv) and is incorporated in what I am calling Theory(iv) which also encompasses Electromagnetic theory. In PRel he developed (iv) which is the "simplest" because no differential equations have to be solved. The concept "impetus" plays a key role in enabling ANW to unify gravitation and Electromagnetism in one theory. I am not certain whether he checked that Theories (ii) and (iii) predict correct values for the two "classical" observations.

Part II sets out in considerable detail concrete physical consequences of Theory(iv). Throughout his mathematical arguments ANW displays extraordinary control of Lagrangian physics, inventiveness, and diligence.

Part III can be described quickly. It is a straight-forward exposition of Tensor Calculus displaying it as a simple algebraic device for discussing a mathematical system which is invariant under the action of a specified group of linear transformations. For example: an orthogonal, a Lorentz, a Symplectic or the full linear group of GTR.

ANW felt this was needed since most physicists in 1922 knew nothing about Tensors and even were frightened by the thought of having to learn Tensor Calculus in order to understand Einstein's theory which, in 1920, seemed esoteric and impenetrable to most physicists. There is an amusing allusion to this on p.182 of CN.

**2.5. IV. The criticism of Einstein's theory by ANW.** This has already been mentioned and is expressed in various contexts in PRel and CN. It is essentially as follows:

- by identifying Whitehead's *potential mass impetus*,  $dJ$ , such that  $dJ^2 = J_{ij}dx^i dx^j$  for an interval  $dx^i$  of the world-line of a particle, with Einstein's *distance*  $ds$  such that  $ds^2 = g_{ij}dx^i dx^j$ , "Einstein leaves the whole antecedent theory of measurement in confusion" (PRel, p.83).

Einstein bases GTR on a vicious circle rather like "which came first, the chicken or the egg?". The confusion can be illustrated in different ways. For example:

**A.** You cannot solve Einstein's equations,  $G_{ij} = 0$ , for  $g_{ij}$  until you enter the initial conditions and *specify your choice of units*. But you cannot explain your choice of unit of length until you solve the equations and give a precise meaning to  $ds^2$  because you began by assuming arbitrary coordinates without defined units.

**B.** The idea of curved space and measurement in GTR is often<sup>13</sup> glossed over with the charming story of The Student, The Ant and The Apple. The 2-dimensional ant walks in a straight line from A to B on the surface of the Apple, then to C and finally to A. He counts and records the number of ant-steps in each side of the triangle ABC. He repeats this for a continuous infinity of triangles and publishes a "Map of my Vicinity Recorded in Units of *Ant-steps*". It was generally agreed that "straight line" should be understood to mean "geodesic". To carry out his program, the ant would need an intuitive ability of recognizing the direction of the tangent to the geodesic at of its each points, in other words of solving the equations of a geodesic. Since no ant is known with this ability nor are "nt-steps" a useful unit we agree that this gargantuan effort of the ant could give no evidence about the curvature, if any, of the apple surface.

Nor do I know any astronomer who claims to have the ability at any point in ST of recognizing the direction of the tangent to the geodesic through the given point to every other fixed point in ST. So, for example he cannot tell how far the earth is from the sun.

This is such a simple clear vicious circle I can see no way out of it. Yet I have never seen it directly addressed by the GTR enthusiasts among whom I must place my younger self! Hitherto I have been a coward unwilling to contradict what *everyone* "knows" is firmly established. Also, until recently, I had not understood how we were misled by Eddington.

**2.6.**

**2.7. V. Eddington's Theorem.** In 1924, one year after his famous treatise on

General Relativity appeared, Eddington published a one-page note in NATURE v.113, proving that, for a spherical static field surrounding a single particle of

mass  $m$ , the metrical field  $g_{ij}$  specified by the Schwarzschild solution of Einstein's equations is isomorphic to the gravitational field  $J_{ij}$  described by ANW's Theory(iv) *except* on the variety  $2m = r$ .

*This theorem shows that Einstein's formula for the Advance of the perihelion of Mercury can be justified by ANW's consistent Theory(iv) and that curved ST is not needed to do so!*

The proof of this key result can be found of course in NATURE, but also, more relevantly<sup>10</sup> for what follows, in the second of the three Lectures by Synge, in 1951 at the University of Maryland. These lectures are so valuable that I made them available in arXiv, Physics/0505027. The main content of that paper to which I refer as "JLS" is Synge's elegant summary of much of the mathematical core of Part II of PRel. It also contains an Introduction by myself in which is embedded a precursor of the present article, and an Appendix, concerning the Solar Limb Effect, discussed briefly below which is appended to this article.

It also suggests that any result claimed by GTR can be justified by PRel without assuming curved ST if the Schwarzschild field is the only aspect of GTR, beyond Newton, required in its alleged "proof". Possibly the results of Probe B will be such a case.

The results of Probe B should be compared with predictions of Theory(iv) and the alleged conclusions of GTR.

**2.8. VI. Solar Spectral Shift.** This has a long and complex history. In 1907 what came to be named *The Limb Effect* was noticed. Because of cancellation of Doppler effect at the ends of an equatorial diameter of the sun's disc we expect the average of frequency of the same line at the two ends to equal that of the line at the center of the disc. The Swedish astronomer, Dr. J.K.E. Halm<sup>14</sup>, working in South Africa, was surprised that he found very few lines which satisfied this expectation. The difference between the expected and observed frequencies from line to line appears almost random.

According to my reading of the literature, this has had no satisfactory explanation after more than a century.

In PRel Chs.10,11, 13-16, Whitehead presents his theory of the gravo-electro interaction which does predict a Limb Effect! In 1946 when I still considered GTR might be a reasonable theory, I drafted a paper examining the consequences of his theory in the hope that it would resolve the advantages of Einstein's versus Whitehead's theories. It was accepted by Evershed, Editor of the Astrophysical Journal, but because of change of jobs I did not manage to submit it in what I considered decent shape. Later in revised form it was accepted for a Russian journal but the Journal never appeared! That paper now appears here as the Appendix.

I consider it very important that this be updated by detailed study of multiplets along the lines initiated by Miss Adam which may be the key to unravelling the complexity of the Solar Spectrum.

This is important for our understanding of both movements of gas in the solar atmosphere and also for the interpretation of the spectra of extra-galactic stars and thus for all our cosmological speculations.

**2.9. \.**

## 2.10. VII. Some Key Personae in the Drama. *In this Section I assemble*

*a few relevant considerations for which there seemed no natural point-of-entry above.*

### 1. **John Keats**, the poet.

"Beauty is Truth and Truth is Beauty,  
That is all ye know on Earth and all ye need to know."

This famous profession of faith of Keats is especially attractive to young mathematicians when they realize how wonderfully simple GTR actually is. Certainly that is how I felt.

We believed that since GTR is so simple it must be true! Perhaps it was this that prevented us and many others from seeing the obvious validity of ANW's Criticism of GTR.

### 2 **Eddington**

It is not widely realized that the existence of Dirac's equation for Hydrogen was a big shock for A.S.Eddington(ASE). It shattered his firm belief that Tensor Calculus is totally sufficient for discussion of Relativity. This led him to develop an elaborate theory about *The Constants of Nature* by which he predicted the values of ten constants. By a method not available to Einstein, ASE used three of these to establish Units and obtained all the remaining with remarkable accuracy. No physical theory has claimed such wide-ranging objective confirmation. But it was dismissed as nonsense by most physicists of the thirties as *philosophical speculation* by a man in his dotage!! For example such was the attitude of Heisenberg with whom I spent two evenings in his apartment in Goettingen in 1946.

Eddington fixed on the fact that the four matrices, used by Dirac to formulate his famous equation, generate a 16-dimensional Algebra<sup>15</sup> over the complex numbers which we now call a Clifford Algebra. ASE attributed such a 16-dimensional algebra to each electron and each proton to carry its *internal structure* which may thus be considered a major generalization of "spin". The theory was developed in a long series of papers which were summarized in two<sup>16</sup> major monographs in 1936 and, posthumously, in 1947.

### 3. **Synge and Schroedinger**

In 1952 Synge invited me to give eight lectures to his Seminar in Dublin with himself and Schroedinger, by no means silent, in the front row! These were about my 1943 Toronto Thesis based on ASE's Theory of the Constants of Nature for which Synge had been one of my Examiners. My Supervisor was Einstein's coauthor, Leopold Infeld. Unlike Heisenberg and other physicists with whom I talked neither Schroedinger nor Synge were willing to dismiss out-of-hand the last fourteen years of work of the man who essentially created modern Astrophysics with his book<sup>17</sup> on the *Internal Constitution of the Stars*.

Personally, though there were a few minor and one major mathematical errors in his argument, I gained great respect for Eddington's physical insight and for his desire to stick to physical fact. I have never noticed these virtues in writings about String Theory and related theoretical activity most of which I regard as mathematical romanticizing.

### 4. **Will's Important Paper**

In 1971, Clifford M. Will<sup>18</sup> published a paper which has played a very significant role in determining how the physics community dealt with Whitehead's Theory (iv). This paper was written before Will had finished work on his Ph.D. under the Supervision of Kip Thorne. At the same time he was working on several

papers with Thorne and others. He implies that he had not read Whitehead in any depth. This indeed was confirmed in my mind by his Footnote (10) in which he ascribes to ANW the opposite of my understanding of Whitehead's choice of units. He concluded that for a certain period in the analysis of the motion of tides Theory (iv) predicts a result which is in error by 200 times the estimated possible error and therefore was sure that he had delivered the *coup de grace* to ANW's Theory. This conclusion was accepted in the well-known comprehensive treatise, GRAVITATION, by Charles Misner, Kip Thorne, and John Wheeler (MTW) and/so became part of our accepted Dogma.

In the only critical discussion of Will's paper I have seen Fowler<sup>19</sup> replaces a basic assumption of Will by an alternative which seems reasonable and concluded that Will had over-estimated ANW's error by a factor of 100! I drew Fowler's paper to Clifford Will's attention some years ago and also pointed out that Dark Matter had not been thought of when he and Fowler wrote and that the existence of Dark Matter would change his conclusion. He felt that this had only an insignificant effect on his argument. I also asked him to rebutt Whitehead's Criticism of GTR, but with no success.

More recently<sup>20</sup> Gary Gibbons and Clifford Will have issued a paper with an improved version of the previous paper. They announce four new criticisms of Theory (iv).

I infer from p.84 of PRel that ANW suspected that his Theory (iv) was not Final. This is partly why he offered three alternative possibilities for a gravitational LAW. Having found LAW (iv) which was Lorentz invariant, logically consistent and predicted the observed values for the two bits of evidence on which Eddington had based his advocacy of GTR, he turned his attention to matters of greater interest to himself. Knowing that he had disposed of GTR as a viable theory Whitehead never thereafter referred to it positively as far as I have noticed.

## 2.11. VIII. Concluding Observations

*Anyone acquainted with current cosmological speculation will realize that the implications of my thesis are almost devastating. Since I do not claim to have mastered String Theory or the Standard Model I appeal to many who are more competent than I to work out in detail the consequences of this paper.*

### 1. Humility

Perhaps the most important is the realization that humanoids must humbly accept the fact that we cannot formulate in human language a "Final Theory". The presumption that we could is essentially why Adam was thrown out of Eden! Personally I have always preferred Whitehead's step-by-step approach to understanding physics than Einstein's exaggerated hopes and claims.

### 2. Theory (iv) is currently the best

At present there is no valid evidence that ST is curved but rather the opposite. As Whitehead noticed between 1917 and 1920, *any theory of gravitation which makes the metric of Space-Time dependent on the circumambient masses is not viable.*

Einstein became famous and GTR thereafter dominated our thinking because of the claim that GTR predicted the two Classical Tests. In fact, the observed results were predicted not by GTR but what I now call the "mongrel" version of GTR

which implicitly assumes that ST is flat. ANW's consistent Theory (iv) explicitly makes the same predictions. In my lectures about GTR, following Eddington and most textbooks I justified the mongrel theory by arguing that locally the gravitational fields are so weak that the Einsteinian and Newtonian metrics could not be observationally distinguished. This may be true "observationally" currently but there is a distinction or the earth would not move in an ellipse! But this justification fails completely for the strong fields which increasingly are of cosmological interest.

In my opinion, Theory (iv) will probably be easier to reconcile with QM and it is not based on a vicious circle. So currently it is the best available theory even though, thanks to Clifford Will, we know that as Whitehead expected it is not Final.

Eddington in connection with his Theory of the Constants of Nature, faced and solved the problem of units. His was a wide-ranging Theory which predicted more than ten quantities. By choosing three determined by functionally independent formulas and setting them equal to accurately observed values he was able to establish meaningful units of length, time and mass. So equipped, he could predict seven Constants with extraordinary accuracy.

It seems to me that at present the most useful expenditure of time and mental energy would be to tackle the problem of reconciling ANW's Theory (iv) with QM. Possibly a first step would be to pursue the problem introduced in the Appendix. Or to consider whether ANW's formula for the gravo-electro Impetus can be modified with advantage by ideas from Quantum Field Theory and/or ANW's metaphysics.

### 3 Extend ANW's Unified Theory

PRel already includes gravitational and electromagnetic forces. in the definition of Impetus Potential in Equation (9) .p. 79. Can we obtain analogous terms for Weak and Strong forces? This would provide a Unified theory of forces now known.

### 4 Important Research Problem

It could be of considerable significance if the research begun in the Appendix were checked and completed. This would entail a detailed study of the variations of frequency of lines of multiplets in the solar spectrum. As observed by Miss Adam<sup>21</sup> of the Oxford Observatory, the Limb Effect for different lines in the multiplet vary in an astonishing manner. It would be desirable to verify, extend and explain her observations.

It seems to me that the observed differences are caused only by gravitational or possibly pressure effects. That is, they provide a considerable body of data for which it might be relatively easy to obtain a convincing explanation. Armed with this we might be able to understand the total spectrum with more assurance than presently is the case.

This could help us to interpret the motion of gases in the Sun's atmosphere and also the spectra of distant stars on which now much cosmological speculation rests.

### 5. Back to Eddington?

It may be worthwhile to return to Eddington's Theory, seeking to understand how he obtained such remarkable predictions

### 6. Revisit Black Holes with ANW?

It may well be very interesting to develop the theory of Black Holes using ANW's theory as set forth by Synge in the Proc.of the Royal Society<sup>22</sup> and in the last few pages of his Maryland Lectures<sup>10</sup>. ANW's treatment of the field of an isolated massive point-particle does not have the Schwarzschild singularity.

## 2.12. IX. APPENDIX. WHITEHEAD' S PERTURBATION OF ATOMIC ENERGY LEVELS

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Whitehead's theory of relativity implies that there is an interaction between the gravitational and electromagnetic fields such that for an atom at the surface of a star, the Coulomb potential  $r^{-1}$  between two charges must be replaced by

$$((1)) \quad \frac{1}{r}(1 - \alpha \cos^2 \theta).$$

Here,  $\theta$  is the angle between the radius vector joining the two interacting charges and the direction of the stellar radius passing through the atom;  $\alpha$  is a constant depending on the strength of the gravitational field. At the surface of the sun,  $\alpha = 2.12 \times 10^{-6}$  approximately.

The effect of (1) is to perturb the normal energy levels by the small term

$$((2)) \quad -\alpha \frac{\cos^2 \theta}{r}$$

which has axial symmetry about the stellar radius through the atom. An effect of precisely this symmetry is what is needed to explain the limb-effect in the solar spectrum. One might also hope that this perturbation could account for the striking differences which have been observed in shifts within the same solar multiplet.

The effect of the perturbation (2) acting between all pairs of charge is to add

$$((3)) \quad V' = \sum_i \alpha \frac{Ze^2}{r} \cos^2 \theta_i - \sum_{i < j} \frac{\alpha e^2 \cos^2 \theta_{ij}}{r_{ij}}$$

to the potential in Schroedinger's equation. Here,  $Ze$  is the charge of the nucleus;  $1 < i, j < N$ , where  $N$  is the number of electrons in the atom;  $\theta_i$  is the angle between  $\mathbf{r}_i$  and the "vertical" ;  $\theta_{ij}$  is the angle between  $\mathbf{r}_{ij}$  and the vertical.

By first-order perturbation theory, the shift in energy of a J,M level is

$$((4)) \quad \Delta E_{JM} = \langle JM | V' | JM \rangle$$

For a Term with total orbital momentum  $L$  and spin  $S$ ,

$$((5)) \quad |JM\rangle = \sum_{\mu+\nu=M} \langle L\mu S\nu | JM \rangle \varphi_{L\mu} U_{S\nu}$$

where  $\langle L\mu S\nu | JM \rangle$  is the vector coupling coefficient, and  $\varphi_{L\mu}$  and  $U_{S\nu}$  are, respectively, the appropriate pure orbital and pure spin functions. Since

$$((6)) \quad \cos^2 \vartheta = \frac{1}{3} + \frac{1}{3}(3 \cos^2 \vartheta - 1)$$

by using the indistinguishability of the electrons, the perturbation (4) can be expressed in the form

$$((7)) \quad \Delta E_{JM} = \langle JM | V_0 | JM \rangle + \langle JM | V_2 | JM \rangle,$$

where

$$((8)) \quad V_0 = \frac{\alpha}{3} N \left( \frac{Ze^2}{r_1} - \frac{N-1}{r_{12}} e^2 \right), \quad \text{and}$$

$$V_2 = \frac{\alpha}{3} N e^2 \left[ \frac{Z}{r_1} (3 \cos^2 \vartheta_1 - 1) - \frac{N-1}{2r_{12}} (3 \cos^2 \vartheta_{12} - 1) \right] \quad (9)$$

The advantage of this decomposition is that with respect to simultaneous rotation of all electrons about the nucleus,  $V_0$  and  $V_2$  belong to  $D_0$  and  $D_2$  representation of the rotation group, respectively.

An application of the Wigner-Eckhart theorem leads to the conclusion that

$$((10)) \quad \Delta E_{JM} = A_L + \frac{3M^2 - J(J-1)}{J(2J-1)} B_J$$

where

$$((11)) \quad A_L = \langle \varphi_{LL} | V_0 | \varphi_{LL} \rangle, \quad B_J = \langle JJ | V_2 | JJ \rangle.$$

By employing (5) and the theory of vector-coupling coefficients, a rather tedious calculation results in the formula

$$((12)) \quad B_J = \sum_{\mu+\nu=J} |\langle L\mu S\nu | JJ \rangle|^2 \frac{3\mu^2 - L(L+1)}{L(2L-1)} B_L = \left\{ 1 + \frac{3(J-L-S)(J+S-L+1)[(J+L-S+1)(J+L+S+2) - 2J-3]}{L(2L-1)(2J+2)(2J+3)} \right\} B_L$$

where

$$((13)) \quad B_L = \langle \varphi_{LL} | V_2 | \varphi_{LL} \rangle.$$

It follows from the Virial Theorem that

$$((14)) \quad A_L = -2\alpha E_L$$

where  $E_L$  is the total energy of the state  $\varphi_{LL}$  which is given with sufficient accuracy for the present purposes by the mean observed energy of the Term. Thus for a Fraunhofer line, the  $V_0$  term gives rise to a red-shift which is proportional to the wavelength of the line and equal to 2/3 of that predicted by Einstein.

We have thus reduced the problem of calculating the Whitehead shift in the levels of a Term to that of evaluating the one constant  $B_L$ . Consequently, the shifts in the lines of a multiplet depend on two constants  $B^i$ ,  $B^f$  associated with initial and final levels.

Dr. R M. Erdahl has suggested that in attempting to check this theory against observations we should treat  $B^i$  and  $B^f$  as phenomenological constants. In certain

cases  $B_J = 0$ , so that for these the predictions are particularly simple. For example, from (13) it follows immediately that  $B_L = 0$  if  $L = 0$ , that is for *an* S-term. But it follows from (12) that  $B_J$  also vanishes for states such as  $^4P_{1/2}$ ,  $^6D_{1/2}$ , ...  $^{10}F_{11/2}$  and many others. It may also be worth looking at Terns for which  $B_J$  is small.

To test the usefulness of Whitehead's perturbation in explaining the actual complex observations of shifts in the solar spectrum, it would be particularly valuable to have reliable measurements for the absolute shifts at various points in the solar disc for all lines of a multiplet and especially for multiplets which include one or more transitions between energy levels with symmetry type appearing in the list described above.

In addition to possible perturbation of energy levels by a gravo-electric interaction, the Fraunhofer lines are undoubtedly shifted by Doppler and pressure effects. To this must be added the classic Einstein shift which has been confirmed by the Pound-Rebka experiment and which follows from Newton's theory and the conservation of energy. The Einstein and Doppler shifts are proportional to the wave-length of the line and by themselves certainly cannot explain the observed shifts in the solar spectrum.

If Whitehead's perturbation combined with reasonable assumptions about pressure shift is unable to explain the observations, all is not lost. If the astronomers can obtain reliable observations, especially at the limb, of a large number of multiplets of diverse symmetry, it should be possible, using the techniques of the present paper, to obtain a good approximation for a perturbation of atomic energy levels which would explain the observations by employing a multipole analysis.

Since in the solar spectrum, the observed deviations from Einstein's predicted shift are as large or larger than his prediction, it is clearly of great interest to establish the source of this deviation in order to be able to interpret spectral shifts from other stars with any confidence.

August 16, 1968.

(NOTE. This paper had a really strange history! In a letter dated September 10, 1948, a first draft was recommended to J.E. Evershed, subject to revisions, for publication in the Astrophysical Journal by H.H. Plaskett, Director of the Oxford Observatory. Though I presented its ideas at a couple of conferences, because of changing professional duties only twenty years later was it revised to its above form.

For many years, I assumed that this paper had appeared in the Proceedings of the International Conference on Relativity and Gravitation in the USSR which R. M. Erdahl and I attended in 1968 and where it was delivered and accepted. Only in 2003, when my old interest in Whitehead's theory was reviving, did my friend Prof. V. I. Yukalov inform me that the Proceedings of the Conference were never published. AJC ).

## 2.13. X. Notes. 1. Trilogy, Cambridge University Press

- 1919 An Enquiry Concerning the Principles of Natural Knowledge (PNK).
- 1920 The Concept of Nature(CN).
- 1922 The Principle of Relativity(PRel).

2. Albert Einstein, *Zur allgemeinen Relativitaetstheorie*, (On the General Theory of Relativity), 1915, Prussian Academy of Sciences

3. In Lowe's biography of ANW<sup>7</sup> the reader can find bibliographical details of Whitehead's published works, pp.367/373 of Volume II, and in Vols. I&II background information for the following selection of items:

(i) 1905 was the year in which ANW submitted *On Mathematical Concepts of the Material World* to the Ph.Trans.of the RS,London,Ser.A,205(1906) 465-525

(ii) *Space, Time and Relativity*, Proc.Aristotelian Soc., N.S. 16 (1915-1916): 104-29

(iv) *Einstein's Theory: An Alternative Suggestion* Times(London) Ed.Supp. Feb. 12,1920, p. 83. This brief note together with Ch.8 of CN convince me that ANW had realized by the end of 1919 that GTR was *logically* non-viable and that he had the main contents of PRel in his head!

(v-vii) The Trilogy, cf. Note(1)

(viii) *Science and the Modern World*, Macmillan,1925

(ix) *Process and Reality -An Essay in Cosmology*, Macmillan, 1929.

4. H. Keeton makes the same point in the book edited by Timothy Eastman and himself: *PHYSICS and WHITEHEAD - Quantum Process and Experience*, 2004, State University of New York. ISBN 0-7914-5913-6 +paperback.

5. Reinhold Niebuhr was one of the most important American protestant theologians. His attitude to the task of the Christian Church in the USA was basically changed in the 1930's by the suffering of members of the working-class in his Parish and the apparent irrelevancy of the Church's message to them. He called for a radical "trans-valuation of values". Scientists like myself often rightly excoriate so-called "fundamentatlists" for being too slow in absorbing Niebuhr's message but we are as bad or worse in showing little signs that we have absorbed ANW's wisdom of almost a century ago!

6. As a teen-ager I ate up Eddington's marvelously clear expositions of GTR and QM. So when in a second-hand book-store, between my junior and senior years undergraduate Course in Toronto U., I saw a book for \$2.50 called *The Principle of Relativity*. I picked it up eagerly assuming it was about GTR!. While Whitehead, the author, praised Einstein to the skies I was surprised that he pointed to a logical flaw in GTR. His criticism has haunted me therefore since I was 20 years old!

7. Victor Lowe's two-volume biography, *ALFRED NORTH WHITEHEAD - The Man and His Work*, 1985 & 1990, The Johns Hopkins Universtiy Press, is regarded as definitive. It certainly provides a valuable detailed Bibliography which I found very useful.

8. I discovered that a vivid, dramatic and amusing portrayal of the intensity of the Einstein euphoria results from skimming through on the Web the first-hand accounts in the NY Times of Einstein's visit to the USA in 1920.

9. "disdain" is not too strong a word for the feeling that I often sensed among hard-nosed physicists who for several decades seemed to regard themselves as the only percipients of Truth. Though, fortunately, the following attitude is not common, it contains such an expressive turn of phrase I put it on record. An Australian friend of mine obtained a Ph.D. in Life Sciences. At one point he sought advice from his Supervisor. "Would it be worthwhile for me to study the Philosophy of Organism propounded by the philosopher Whitehead?"... "For Heaven's sake,

NO! You are surely old enough to realize that Philosophy is nothing but mental masturbation.”

10. The lectures of J.L. Synge on Dynamics, Tensors and GTR were the most precise and elegant mathematical lectures I ever attended. His two-volume treatise on Relativity: Special, 1955, General, 1960 was characterized by an astrophysicist friend as the best currently available. In 1952 when he invited me to lecture to his Seminar at the Dublin Institute we became and remained friends until his death in 1995. Even so I regret that his failure to understand Part I of PRel and his penchant for geometrizing all of physics helped to prevent us from grasping the truth of PRel. In fact he came close to seeing the point with his remark, p.296/7 of his book on GTR, that Einstein’s method of proving the validity of his prediction of the Advance of the perihelion of Mercury was ”intellectually repellent”! This came close to saying that the prediction was made by a ”mongrel” theory.

11. Dean R. Fowler presented a Ph.D. thesis in Theology of which the first part contains an insightful account of ANW’s approach to Physics. In writing this he had assistance from an competent theoretical Physicist. This is summarized in *Process Studies*, 5, 159-174(1975). Based on this he criticized Will’s article, asserting that its estimate of ANW’s error was 100 times too large, *ibid.* 4,4(1974). More recently, the philosopher Gary Herstein, has published a book, setting forth in some detail the criticism of GTR by ANW: Herstein, Gary L. *Whitehead and the Measurement Problem of Cosmology*, Ontos-Verlag, Frankfurt / Lancaster / New Brunswick, Process Thought V, June 2006.

12. In section 42 of the *Dialogues of Alfred North Whitehead*, by Lucien Price, of which there have been several editions including paperback, Whitehead states in forceful language his conviction that humanoids will never be able to fashion a ”final” theory.

13. This cute story is employed, for example, in the widely used textbook GRAVITATION by Misner, Thorne and Wheeler.

14. The discovery of the Limb Effect is attributed to the Swedish astronomer, J.K.E. Halm, who was Chief Assistant at the Cape Observatory in South Africa from 1907 to 1927. It is believed that in 1907 he and the Astronomer, Hough were observing the frequencies of lines from various points on the solar disc.

15. In analogy with Dirac’s equation, to a basis ( $e_i$ ) of 4-dimensional Euclidean space, Eddington associated four  $4 \times 4$  matrices ( $E_i$ ) such that

$$E_i E_j + E_j E_i = 2\delta_{ij} I$$

where  $I$  is the identity operator. By taking all possible products of the  $E_i$  then by using the above anticommutation properties we find that there are 16 linearly independent products which span a 16-dimensional linear space over the complex numbers. This space can be regarded as a direct sum of the 6-dimensional subspace of skew-symmetric matrices and the 10-dimensional space of symmetric matrices. ASE attributed the electromagnetic properties of the particle to the first of these subspaces of dimension 6 and the mechanical properties to the symmetric sub-space. Analogously the internal properties of two particles requires a linear space of dimension  $16 \times 16 = 256$  dimensions with two subspaces of dimension of 120 and 136. By more than one argument, the first in 1931, ASE was led to consider the equation

$$10m^2 - 136mm_0 + m_0^2 = 0$$

with roots of ratio 1847.60 which is so close to the ratio of mass of proton to electron that ASE, the physicist, could not ignore it. Until his death in November 1944 his intense research effort was devoted to understanding this and several other numbers to which he was led.

16. *Relativity Theory of Protons and Electrons*, Cambridge University Press, 1936 (RTPE); *Fundamental Theory*, ibid, 1946 (FT)

Interested readers might begin with the Preface of FT and the Table on p.66.

17. *The Internal Constitution of the Stars*, Cambridge University Press, 1930.

18. Clifford M. Will, *Astrophysical Journal*, 169,141-155(1971)

19. *Process Studies*, 4,4

20. Gary Gibbons and C.M. Will, *arXiv:gr-qc/0611006v1* 1 Nov 2006.

21. Madge Adam, *Mon. Notices, RAS*,119,460-470(1959); ibid.177,687-707(1976).

In these and other papers she reports shifts for a variety of multiplets.

22. J. L. Synge, *Proc. Roy. Soc.*, London A211(1952) 303.

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